Phalanx Duel Classic v3.0

Formal Mathematical, Algorithmic & Verification Specification

Auto-bound to `engine/src/combat-math.ts`, `engine/src/state-machine.ts`, `engine/src/liveness.ts`, and `docs/gameplay/rule-evidence.json` (71 normative rules · 63 aligned · 7 partial · 1 unverified).

1 State Space & Geometry Invariants

A Phalanx Duel match is a finite-state, zero-RNG-after-deal deterministic transition system $\mathcal{M} = (\mathcal{S}, \mathcal{A}, \delta, s_0, \mathcal{E})$ parameterized by a rectangular battlefield of r rows and c columns (Classic: $r = 2, c = 4$, starting Life Points $\mathrm{LP}_0 = 20$). Each player $p \in \{0, 1\}$ owns an independent 52-card deck D_p shuffled deterministically from a recorded seed.

> **Axiom 1 — Global Geometry & Scarcity Invariants (PD-RULE-005..008)**
>
> For every legal parameter tuple $(r, c, h_{\max}, D_0)$:
>
> $$r \cdot c \leq 48, \quad h_{\max} \leq c, \quad D_0 = rc + c, \quad 52 - D_0 \geq 4$$
>
> In Classic v3.0 ($r = 2, c = 4$): $rc = 8$ battlefield slots, $h_{\max} = 4$ hand cards, initial draw $D_0 = 12$, leaving a deterministic drawpile reserve of $52 - 12 = 40 \geq 4$ cards.

2 Deterministic Turn Lifecycle

The authoritative transition function $\delta : \mathcal{S} \times \mathcal{A} \to \mathcal{S} \times \mathcal{E}^*$ advances through an eight-phase finite state machine verified by `scripts/ci/verify-doc-fsm-consistency.ts`:

$$\ldots\mathrm{rtTurn} \to \mathrm{DeploymentPhase} \to (\mathrm{AttackPhase} \to \mathrm{AttackResolution} \to \mathrm{CleanupPhase} \to \mathrm{ReinforcementPhase} \to \mathrm{DrawPhase} \to \mathrm{EndTurn})^* \to \mathrm{gameC}\ldots$$

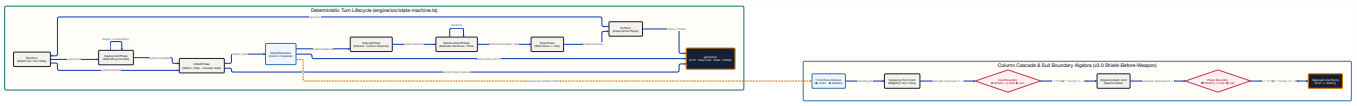


Figure 1: Single-source D2 diagram of the 8-phase FSM (`engine/src/state-machine.ts`) and v3.0 Column Cascade (`engine/src/combat-math.ts`).

3 Combat Cascade & Suit Boundary Algebra

When a front-rank attacker a with base attack d_0 strikes column j, damage propagates deterministically through front card f, card-to-card boundary $\mathcal{B}_{\mathrm{card}}$, back card b, player boundary $\mathcal{B}_{\mathrm{player}}$, and opponent Life Points LP.

3.1 Card Transition Function (`resolveCardTransition`)

Let $h \geq 1$ be the target card's current HP and $d > 0$ the incoming damage.

1. **Face-Card Eligibility Partial Order (`isFaceCardEligible`):** On face-card defenders $\{J, Q, K\}$, face-card attackers obey the rank hierarchy

   $$\mathrm{Jack} \preceq \{\mathrm{Jack}\}, \quad \mathrm{Queen} \preceq \{\mathrm{Jack}, \mathrm{Queen}\}, \quad \mathrm{King} \preceq \{\mathrm{Jack}, \mathrm{Queen}, \mathrm{King}\}$$

   (while non-face attackers—number cards and Aces—are unrestricted by this check). If `modeClassicFaceCards` is active and a `jack` attacks `{Q, K}` or a `queen` attacks `K`, the target is ineligible: $c_{\mathrm{out}} = 0$, $\mathrm{destroyed} = \bot$, and in Classic mode $h' = h$ (in Cumulative mode $h' = \max(h - d, 1)$).

2. **Ace Duality (`modeClassicAces`):** If the target t is an Ace:
   - **Front-rank Ace vs. Ace:** standard lethal absorption applies ($\mathrm{abs} = \min(d, h)$, $h' = h - \mathrm{abs}$, $c_{\mathrm{out}} = d - \mathrm{abs}$).
   - **All other matchups (`aceInvulnerable`):** the Ace absorbs at most 1 damage and cannot be destroyed:

$$\text{abs} = \min(d, 1), \quad h' = 1, \quad c_{\text{out}} = d - \text{abs}, \quad \text{destroyed} = \bot$$

3. **Standard Card Absorption:**

$$\text{abs} = \min(d, h), \quad h' = h - \text{abs}, \quad c_{\text{out}} = d - \text{abs}, \quad \text{destroyed} \Leftrightarrow (h' = 0)$$

3.2 Non-Commutative Suit Boundary Operators

Suits are **boundary-scoped timing operators**, not passive stats. Shields (♦, ♥) fire only when the card carrying them is **destroyed** at that boundary; weapons (♣, ♠) fire only when the **attacker** carries that suit.

Definition 1 — Card-to-Card Boundary Operator (resolveCardBoundary)

Given post-front carryover $c_1 > 0$, destroyed-front Diamond shield value $\delta_{\diamondsuit} \geq 0$, and Club eligibility $I_{\clubsuit} \in \{0, 1\}$:

$$\mathcal{B}^{v3}_{\text{card}}(c_1) = \begin{cases} 2 \cdot \max(0, c_1 - \delta_{\diamondsuit}) & \text{if } I_{\clubsuit} = 1 \text{ and } c_1 > \delta_{\diamondsuit} \\ \max(0, c_1 - \delta_{\diamondsuit}) & \text{otherwise} \end{cases}$$

Definition 2 — Player LP Boundary Operator (resolvePlayerBoundary)

Given post-back (or empty-back) carryover $c_2 > 0$, last-destroyed Heart shield value $\delta_{\heartsuit} \geq 0$, and Spade weapon flag $I_{\spadesuit} \in \{0, 1\}$:

$$\mathcal{B}^{v3}_{\text{player}}(c_2) = \begin{cases} 2 \cdot \max(0, c_2 - \delta_{\heartsuit}) & \text{if } I_{\spadesuit} = 1 \text{ and } c_2 > \delta_{\heartsuit} \\ \max(0, c_2 - \delta_{\heartsuit}) & \text{otherwise} \end{cases}$$

Theorem 1 — Non-Commutativity of Shield and Weapon Operators

Let $S_\delta(x) = \max(0, x - \delta)$ be a boundary shield ($\delta \geq 0$) and $W(x) = 2x$ be a weapon multiplier. For all $x \geq 0$ and $\delta \geq 0$:

$$(S_\delta \circ W)(x) - (W \circ S_\delta)(x) = \max(0, 2x - \delta) - 2\max(0, x - \delta) = \min(\delta, \max(0, 2x - \delta)) \geq 0$$

with strict inequality ($(W \circ S_\delta)(x) < (S_\delta \circ W)(x)$) **if and only if** $0 < \delta < 2x$. **Consequence:** v3.0 applies **Shield before Weapon** ($W \circ S_\delta$), strictly reducing burst variance relative to legacy v1.0 ($S_\delta \circ W$) whenever $0 < \delta < 2x$, and ensuring any shield with $\delta \geq x$ zeroes out the doubling trigger completely.

4 Total Termination (Liveness Theorem)

In `engine/src/liveness.ts`, every v3.0 `AttackPhase` turn boundary evaluates a collision-free canonical position witness $\sigma(s)$ (encoding `specVersion`, `phase`, `activePlayerIndex`, `passState`, and ordered per-player card tokens across hand, battlefield, drawpile, and discard) alongside a four-component progress vector:

$$\Phi(s) = \left(\sum_p \text{LP}_p, \quad \sum_p \sum_k \text{hp}_{p,k}, \quad \sum_p |\text{Draw}_p|, \quad -\sum_p |\text{Discard}_p| \right)$$

Irreversible progress holds iff any coordinate of Φ strictly decreases between turn boundaries.

Theorem 2 — Guaranteed Finite Termination (Liveness Policy v3.0)

For every valid v3.0 action sequence $a_1, a_2, \ldots$, the match reaches `gameOver` in at most 200 completed turns. At every `AttackPhase` turn boundary without a decisive 0-LP or forfeit outcome, draw termination is decided by the strict precedence lattice:

$$\text{count}(\sigma(s)) \geq 3 \Longrightarrow \text{repetitionDraw} \quad \succ \quad \text{noProgressTurns} \geq 50 \Longrightarrow \text{noProgressDraw} \quad \succ \quad t \geq 200 \Longrightarrow \text{turnLimitDraw}$$

Because `turnNumber` increments strictly on every completed turn and `hardTurnLimit = 200`, no infinite game loop is reachable in v3.0.

5 Authority Ledger, Fog-of-War & Cryptographic Replay

Every match is an immutable fold from its initial seed $s_0 = \text{createInitialState}(\text{config})$ over the server-authoritative action log:

$$s_n = \delta^*(s_0, \langle a_1, \ldots, a_n \rangle), \quad h_t = \text{SHA-256}(\text{canonical}(s_t))$$

Each client surface receives only a redacted projection $\pi_{o(s_t)}$ stripping opponent hidden hand and drawpile order; spectators receive $\pi_{\text{spec}(s_{t'})}$ bounded by `replayGameAtOrBeforeTurn` for anti-stream-snipe delay. Any third party can verify match integrity offline via `GET /api/matches/:id/verify` by asserting $H(\text{replayed}) = h_t$ at every turn.

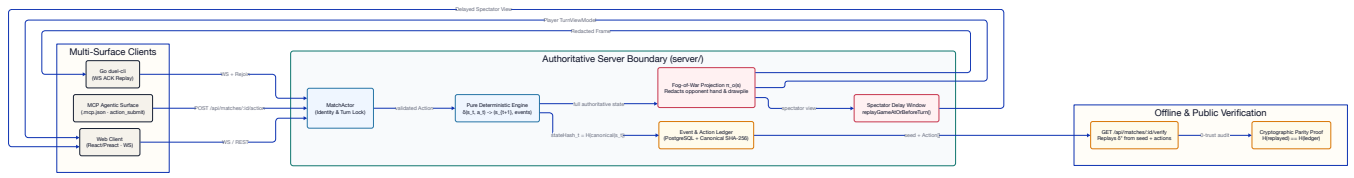


Figure 2: Server-authoritative action ledger, Fog-of-War projection functor π_o, and zero-trust replay verification.

6 Machine-Verified Rule Evidence Matrix

The table below is compiled **directly** from `docs/gameplay/rule-evidence.json` at PDF build time—preventing specification drift between documentation and CI.

Metric	Count	Formalization Type	Rules
Total Normative Rules	**71**	Predicates & Invariants	22
Aligned with Engine + Tests	**63**	Transitions & Protocols	15
Partial Alignment	7	Equations & Inequalities	14
Unverified (`E0`)	1	Evidence Tier `E1+`	68

Formal Verification & Assurance Surface (monograph-metrics.json)

- **Package Version:** `v#metrics.packageVersion` (build `#metrics.buildNumber`) · **FSM:** `#metrics.fsm.phaseCount` phases, `#metrics.fsm.transitionCount` transitions (`#metrics.fsm.nonTerminalPhases` non-terminal, verified deadlock-free).
- **Unbounded SMT-LIB2 Proofs (Z3):** `#metrics.formalSpec.smtProofSuite` — **8 / 8** unbounded theorems proved over all integers $\mathbb{Z}$ (`unsat` Q.E.D. via `pnpm verify:proofs`: Shield-Zeroes-Weapon, v3.0 Burst-Dominance, Exact Non-Commutativity Identity, Strict Non-Commutativity iff $0 < \delta < 2x$, Damage Conservation, Ace Invulnerability Conservation, Liveness Bounded Termination ≤ 200, and Fog-of-War Non-Interference).
- **TLA+ Model & Executable Verifier:** `#metrics.formalSpec.tlaModule` mirrored by `#metrics.formalSpec.verifierScript` (`pnpm verify:formal`).
- **Exhaustive Combat Oracle:** `#metrics.formalSpec.exhaustiveCombatTransitions` verified combat transitions (`pnpm rules:combat-reference`).
- **Liveness Policy:** `#metrics.liveness.repetitionOccurrences`× repetition · `#metrics.liveness.noProgressTurns`-turn no-progress · `#metrics.liveness.hardTurnLimit`-turn hard cap.